Whitman College Econ 328 Final Exam May 10, 2013

Write all of your answers in your blue book. Show all of your work in your bluebook. The exam ends at 4:30.

1. (a) (5pts) Define the prisoners' dilemma class of games.

(b) (5pts) Describe the strategic tension inherent in the prisoners' dilemma class of games.

(c) (5pts) Provide an example of a game in the prisoners' dilemma class. Draw the normal form matrix for your game.

2. (15pts) Give an example of a game that has a pure strategy Nash equilibrium (NE), but no mixed strategy NE. Draw the normal form matrix for this game, indicate the pure strategy NE, and explain why the game has no mixed strategy NE.

3. (15pts) Give an example of a game that has a mixed strategy Nash equilibrium (NE), but no pure strategy NE. Draw the normal form matrix for this game and explain why the game has no pure strategy NE. You do not need to find the mixed strategy NE.

4. For parts (a)-(h) below, consider the following extensive-form game.



- (a) (4pts) List the Pareto efficient payoff profiles for the game.
- (c) (10pts) Write the normal form matrix for the game.
- (d) (3pts) List the set of rationalizable strategies for Player 1.
- (e) (3pts) List the set of rationalizable strategies for Player 2.
- (f) (5pts) Is this game a prisoners' dilemma? Explain your reasoning.

(g) (10pts) Compute the pure strategy Nash equilibria of the game. List these Nash equilibria strategy profiles.

(h) (10pts) Find the subgame perfect equilibria of the game. Show all of your work.

5. The subgame perfect equilibrium (SPE) concept is a refinement of the Nash equilibrium (NE) concept. That is, the SPE takes the set of NE and pares it down to a subset.

(5pts) True or false? If a bad Nash equilibrium exists, the SPE paring process will remove the bad Nash. Explain your answer, including providing a definition of a bad Nash.

6. Consider a situation of strategic interaction between two people, Lance (L) and Bradley (B), who could rent an apartment together. Weekly rent is \$200. The landlord would require each tenant to sign a contract agreeing to be responsible for half of the rent. The apartment has two bedrooms and a shared living room and kitchen. Lance would get the large master bedroom with a view, attached bathroom and walk-in closet. From living in this bedroom and sharing the apartment with Bradley, Lance would get utility of 250 minus the 100 he would pay in rent. Bradley would get a small, unattractive bedroom. From living in this bedroom and sharing the apartment with Lance, Bradley would get utility of 150 minus the 100 he would pay in rent. If they do not come to an agreement to live together, each would get a utility of 20. Assume that utility can be transferred 1:1 from one roommate to another via side payments of money. Lance has a discount factor of 0.86. Bradley has a discount factor of 0.90.

(a) (5pts) If the roommates reach an agreement to rent the apartment together, what is the joint value of the agreement?

(b) (5pts) What is the surplus from the agreement to rent the apartment together?

Suppose that Lance proposes that they rent the apartment together, and makes an offer to Bradley of how to split the surplus between them. Bradley can accept the offer, or he can make a counteroffer of how to split the surplus. Assume that the alternating series of offers could continue indefinitely. Consider the standard bargaining solution for this situation.

- (c) (10pts) What share of the surplus does Lance get?
- (d) (10pts) What utility does each roommate get?
- (e) (5pts) Which, if either, roommate makes a side payment, and what size is that payment?

(f) (10pts) Does Lance gain an advantage from being the first to make an offer? Explain and support your answer.

5. Consider a one-shot tariff-setting game between two countries. Let x_1 be the tariff level of Country 1 and x_2 be the tariff level of country 2. Country 1's payoff is $3000 + 50x_1 + x_1x_2 - (x_1)^2 - 100x_2$. The situation is symmetric, so Country 2's payoff is $3000 + 50x_2 + x_2x_1 - (x_2)^2 - 100x_1$. Assume that tariffs cannot be negative, and that countries set tariff levels simultaneously and independently. Note that the strategy spaces are continuous.

(a) (15tps) Find the best response functions for each country.

(b) (10pts) Compute the Nash equilibrium (NE).

(c) (5pts) Show that if each country chooses a zero tariff level, both would be better off than they are in the NE.

(d) (5pts) Is the NE a bad Nash? Explain your reasoning.

(e) (5pts) Is this game a prisoners' dilemma? Explain your reasoning.

(f) (5pts) Suppose Country 1 chooses a zero tariff level. Find Country 2's best response. What would the payoffs be for each country, if Country 1 chose a zero tariff, and Country 2 took its best response to a zero tariff?

Now consider an infinitely repeated tariff-setting game, where the stage game is the one-shot tariff game described above. Suppose that Country 1 has a discount factor of 0.9, and Country 2 has a discount factor of 0.6.

(f) (15pts) Does the grim trigger strategy support choosing zero tariff levels in each stage as a Nash equilibrium of the supergame? Explain your reasoning.